

Chapter 1: Points, Lines, and Planes

High School Geometry • Chapter 1 • Updated March 2026 • ~40 min read

Learning Objectives

- Identify and describe points, lines, line segments, rays, and planes using correct notation
- Apply the Segment Addition Postulate and the Angle Addition Postulate
- Classify angles as acute, right, obtuse, or straight
- Identify complementary, supplementary, vertical, and adjacent angle pairs
- Use an angle bisector to find unknown angle measures
- Apply properties of perpendicular lines
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1.1 Undefined Terms: Points, Lines, and Planes

Geometry begins with three **undefined terms** — concepts that are fundamental but cannot be precisely defined using simpler terms. Instead, we describe them intuitively.

The Three Undefined Terms

- A **point** has no size — it indicates a location. Labeled with a capital letter: point A .
- A **line** has no thickness and extends infinitely in both directions. Named by two points on it: \overleftrightarrow{AB} , or by a lowercase letter l .
- A **plane** has no thickness and extends infinitely in all directions. Named by three non-collinear points or an uppercase letter: plane ABC or plane P .

From these undefined terms we build **defined terms**:

Segments and Rays

- A **line segment** \overline{AB} consists of endpoints A and B and all points between them. Its length is AB (no bar).
- A **ray** \overrightarrow{AB} starts at endpoint A , passes through B , and extends infinitely in one direction.
- Two rays that share an endpoint and point in opposite directions form a **line**.

Object	Symbol	Read as
Point A	A	"point A"
Line through A and B	\overleftrightarrow{AB}	"line AB"
Segment from A to B	\overline{AB}	"segment AB"
Length of AB	AB	"the distance AB"
Ray from A through B	\overrightarrow{AB}	"ray AB"

The Segment Addition Postulate

Postulate 1.1 — Segment Addition Postulate

If B is between A and C , then $AB + BC = AC$. Conversely, if $AB + BC = AC$, then B is between A and C .

Example 1.1 — Using the Segment Addition Postulate

Points A , B , C are collinear with B between A and C . If $AB = 3x + 1$, $BC = 2x - 4$, and $AC = 22$, find x and BC .

Step 1: Apply the postulate: $AB + BC = AC$

$$(3x + 1) + (2x - 4) = 22$$

$$5x - 3 = 22$$

$$5x = 25 \implies x = 5$$

Step 2: $BC = 2(5) - 4 = 6$

TRY IT

P is between Q and R . $QP = 2x + 3$, $PR = x + 7$, $QR = 25$. Find x and PR .

1.2 Angles and Angle Measurement

An **angle** is formed by two rays (called **sides**) that share a common endpoint called the **vertex**. We write $\angle ABC$ where B is the vertex, or simply $\angle B$ if there is no ambiguity.

Classifying Angles by Measure

- **Acute angle:** $0^\circ < m\angle A < 90^\circ$
- **Right angle:** $m\angle A = 90^\circ$ (indicated by a small square at the vertex)
- **Obtuse angle:** $90^\circ < m\angle A < 180^\circ$
- **Straight angle:** $m\angle A = 180^\circ$ (a straight line)

The Angle Addition Postulate

Postulate 1.2 — Angle Addition Postulate

If point D is in the interior of $\angle ABC$, then $m\angle ABD + m\angle DBC = m\angle ABC$.

Example 1.2 — Applying the Angle Addition Postulate

Ray BD is in the interior of $\angle ABC$. If $m\angle ABD = (2x + 10)^\circ$, $m\angle DBC = (3x - 5)^\circ$, and $m\angle ABC = 90^\circ$, find x .

$$(2x + 10) + (3x - 5) = 90$$

$$5x + 5 = 90 \implies 5x = 85 \implies x = 17$$

Check: $m\angle ABD = 44^\circ$, $m\angle DBC = 46^\circ$, and $44 + 46 = 90^\circ \checkmark$

1.3 Angle Pairs

Special Angle Pairs

- **Complementary angles:** two angles whose measures sum to 90°
- **Supplementary angles:** two angles whose measures sum to 180°
- **Adjacent angles:** two angles that share a vertex and a side, with no overlap
- **Vertical angles:** opposite angles formed when two lines intersect; vertical angles are always congruent
- **Linear pair:** adjacent angles that form a straight line; their measures sum to 180°

Theorem 1.1 — Vertical Angles Theorem

Vertical angles are congruent: if two lines intersect, the opposite (vertical) angles are equal in measure.

Example 1.3 — Finding Angles with Vertical Angles

Two lines intersect forming four angles. One angle measures $(5x + 12)^\circ$ and its vertical angle measures $(8x - 15)^\circ$. Find the measure of each angle.

Since vertical angles are equal: $5x + 12 = 8x - 15$

$$27 = 3x \implies x = 9$$

$$\text{Angle measure} = 5(9) + 12 = \mathbf{57^\circ}$$

$$\text{The adjacent supplementary angle} = 180^\circ - 57^\circ = \mathbf{123^\circ}$$

TRY IT

Two angles are supplementary. One measures $(3x + 20)^\circ$ and the other measures $(x + 40)^\circ$. Find both angles.

1.4 Angle Bisectors

Definition: Angle Bisector

An **angle bisector** is a ray that divides an angle into two congruent angles. If \overrightarrow{BD} bisects $\angle ABC$, then $m\angle ABD = m\angle DBC = \frac{1}{2}m\angle ABC$.

Example 1.4 — Angle Bisector Problem

\overrightarrow{BD} bisects $\angle ABC$. If $m\angle ABD = (4x - 8)^\circ$ and $m\angle DBC = (2x + 16)^\circ$, find $m\angle ABC$.

Since \overrightarrow{BD} bisects, the two angles are equal:

$$4x - 8 = 2x + 16 \implies 2x = 24 \implies x = 12$$

$$m\angle ABD = 4(12) - 8 = 40^\circ$$

$$m\angle ABC = 2 \times 40^\circ = 80^\circ$$

- ◆ **Geometry Tip:** When an angle bisector splits an angle, the two resulting angles are always **congruent** (equal measure). Set the two expressions equal to each other, then solve — don't sum them to get the total angle.

Practice Problems

Problem 1

M is between L and N . $LM = 4x - 2$, $MN = 2x + 8$, $LN = 42$. Find x , LM , and MN .

► [Show Solution](#)

Problem 2

Classify each angle as acute, right, obtuse, or straight:

(a) $m\angle A = 35^\circ$ (b) $m\angle B = 90^\circ$ (c) $m\angle C = 127^\circ$ (d) $m\angle D = 180^\circ$

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Problem 3

Two angles are complementary. One angle is 14° more than three times the other. Find both angles.

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Problem 4

Two lines intersect. One of the four angles formed measures $(6x - 30)^\circ$ and an adjacent angle measures $(2x + 10)^\circ$. These two angles form a linear pair. Find all four angle measures.

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Problem 5

\overrightarrow{KM} bisects $\angle JKL$. If $m\angle JKL = (10x - 20)^\circ$ and $m\angle MKL = (3x + 12)^\circ$, find $m\angle JKL$.

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Problem 6

Name each figure using correct geometric notation:

- (a) A line passing through points P and Q
- (b) A segment with endpoints X and Y
- (c) A ray starting at A passing through B
- (d) The length of segment CD

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Problem 7

Ray \overrightarrow{EG} is in the interior of $\angle DEF$. $m\angle DEF = 124^\circ$ and $m\angle DEG = (3x + 7)^\circ$. If \overrightarrow{EG} bisects $\angle DEF$, find x .

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Problem 8 — Multi-Step

Angle A and angle B are supplementary. Angle B and angle C are complementary. If $m\angle A = 110^\circ$, find $m\angle B$ and $m\angle C$.

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Chapter Summary

BASIC ELEMENTS

Points, Lines, Planes

A *point* has no dimension. A *line* extends infinitely in two directions. A *plane* is a flat surface extending infinitely. These are undefined terms in geometry.

Segments and Rays

A *segment* \overline{AB} has two endpoints. A *ray* \overrightarrow{AB} starts at A and extends through B infinitely. Midpoint divides a segment into two equal parts.

Angle Types

Acute: $0^\circ < m\angle < 90^\circ$. Right: 90° . Obtuse: $90^\circ < m\angle < 180^\circ$. Straight: 180° .

Special Angle Pairs

Complementary: $\text{sum} = 90^\circ$. Supplementary: $\text{sum} = 180^\circ$. Vertical angles: formed by intersecting lines, always congruent.

KEY POSTULATES

- Segment Addition Postulate:** If B is between A and C , then $AB + BC = AC$
- Angle Addition Postulate:** If D is in the interior of $\angle ABC$, then
$$m\angle ABD + m\angle DBC = m\angle ABC$$
- Unique Line Postulate:** Through any two points, exactly one line exists

Key Terms

Collinear

Points that lie on the same line.

Coplanar

Points or lines that lie in the same plane.

Midpoint

The point that divides a segment into two congruent parts.

Angle Bisector

A ray that divides an angle into two congruent angles.

Vertical Angles

Non-adjacent angles formed by two intersecting lines. Always congruent.

Linear Pair

Two adjacent angles that form a straight line. Always supplementary (sum = 180°).